Mars Orbiter Laser Altimeter: Receiver Model and Performance Analysis

James B. Abshire
Xiaoli Sun
Robert S. Afzal

NASA Goddard Space Flight Center
Laser Remote Sensing Branch, Code 924
Greenbelt, Maryland 20771

(301) 614-6717
james.abshire@gsfc.nasa.gov

August, 1999

Abstract

The measurement approach, receiver design, data conversion and calibration of the Mars Orbital Laser Altimeter (MOLA) are described. The MOLA measurements include the range to the surface, which is determined by the laser pulse time-of-flight, the surface slope determined by the received laser pulse width, and the surface reflectivity determined by the ratio of the transmitted and the received laser pulse energies. The instrument performance is analyzed for these measurements.

1 Introduction

The Mars Orbiter Laser Altimeter (MOLA) [1, 2, 3] is one of the four instruments on board NASA's Mars Global Surveyor (MGS) spacecraft [4]. Figure 1 shows a sketch of MGS spacecraft and MOLA instrument. MOLA measures the distance from the MGS spacecraft to the Mars surface by measuring the time-of-flight of its laser pulses. The topographic height of the planet's surface at the laser footprint spot is then determined through the geometry of the planet radius, the spacecraft orbit altitude, and the pointing angle of the instrument. A simplified measurement geometry is shown in Figure 2 and the specifications for MOLA are given in Table 1..

The accuracy of the surface height determination is governed by uncertainties in the time of flight measurement, the spacecraft position, and the pointing angle of the laser beam. The range from MGS spacecraft to the target is related to the laser pulse time-of-flight by

$$R_m = \frac{c\, T_{opt}}{2} \tag{1}$$

with $c = 299792458 m/s$ the vacuum speed of light [5]. Here we have neglected the effect of atmosphere path delay, which is only a few centimeters on Mars due to the low (4-6mbar) surface pressure. The topography or the surface height at the laser footprint can be written as

$$h_s = \left[R_{MGS}^2 + R_m^2 - 2R_m R_{MGS} cos(\phi)\right]^{1/2} - R_{ref} \tag{2}$$

where R_{MGS} is the radius of the MGS spacecraft, ϕ is the pointing angle with respect to nadir, and R_{ref} is the reference surface of the planet which is often taken to be the geoid.

For measurements from space, a small laser beam pointing uncertainty can cause a sizable difference between the actual and the predicted laser footprint location on the ground. The resultant ranging error due to pointing uncertainty is given as [6, 7]

$$\sigma_\phi \approx R_m \left[tan^2(\phi + \theta_\|) Var(\Delta\phi_\|) + \frac{tan^2\theta_\perp cos^2\theta_\| cos^2\phi}{cos^2(\phi + \theta_\|)} Var(\Delta\phi_\perp)\right]^{1/2} \tag{3}$$

where $\theta_\|$ and $\theta_\perp$ are the effective surface slopes parallel and perpendicular to the plane of the expected laser beam and nadir axes, and $\Delta\phi_\|$ and $\Delta\phi_\perp$ are the pointing uncertainties in the two directions.

The MGS spacecraft pointing knowledge is less than 1 mrad after post navigation signal processing [8]. This gives a range uncertainty of 7 meters for a 400km average range, nadir pointing, 1 degree surface slope, and the pointing error in the same direction as the surface slope. Since the pointing error is a slowly varying random variable, it usually spatially displaces the footprint location in the ground track and has little effect on the measurements of local topography features.

In addition to the laser pulse time-of-flight, MOLA also measures the transmitted and the echo laser pulse energies and the echo pulse width at the threshold crossings. The full echo pulse energy and the rms pulse width can be solved as long as the pulse shape is known.

The pulse energies can be used to determine the surface reflectivity by using laser altimeter link equation

$$E_r = E_{tr}\tau_r \cdot \frac{A_r}{R_m^2} \cdot \frac{r_s}{\pi} \cdot \tau_a^2 \tag{4}$$

where

E_r: received signal pulse energy (Joule),
E_{tr}: transmitted laser pulse energy (Joule),
τ_r receiver optics transmission,
A_r: receiver telescope entrance aperture area (m^2),
r_s: target surface diffusive reflectivity,
τ_a: Mars atmosphere transmission, one way.

The echo pulse width can be used to estimate the surface slope and roughness within the laser footprint. The surface slope of Mars is usually much larger than the spacecraft off-nadir pointing angle. If roughness is neglected, the rms pulse width of the echo laser pulse is related to the surface slope as [6]

$$< \sigma_r^2 >= (\sigma_x^2 + \sigma_f^2) + \frac{4R_m^2}{c^2}\left[tan^4(\gamma) + tan^2(\gamma)tan^2(\theta)\right], \tag{5}$$

where σ_x is the transmitted laser rms pulse width, σ_f is the rms receiver impulse response, and γ is the rms laser beam divergence angle (half angle at the $1/\sqrt{e}$ intensity point). The first term in the bracket of Eq. (5) accounts for the laser beam curvature effect and can often be neglected since usually $\gamma << \theta$.

The laser pulse time-of-flight, the echo laser pulse energy and rms pulse width, can be determined from the MOLA measurements by using the procedures outlined in the rest of this paper under the following assumptions: the transmitted and the received pulse shapes are Gaussian and the instrument receiver is at its nominal operating temperature. On flight measurement data indicate that there have been no significant MOLA receiver changes since its pre-launch calibration.

2 Laser Transmitter

The MOLA laser transmitter is a diode-laser pumped, Q-switched, Cr:Nd:YAG slab laser. The laser design details and a performance model is given in [9]. The nominal transmitted pulse energy is 45 mJ and the pulsewidth is 8 ns full width at half maximum (FWHM). The laser beam full divergence angle was measured to be 0.42 mrad at the 90% encircled energy in the far field. Assuming Gaussian beam profile, the corresponding rms beam divergence angle was 0.093 mrad.

A small amount of the transmitted laser light is collected and coupled into the start pulse photodetector via a multimode optical fiber. The transmitted pulse energy is measured by the start pulse energy counter, which consists of a charge to time converter and an 8 bit counter. A lowpass filter is used to broaden the pulse for the comparator circuit to trigger reliably. The filter has no effect on the total pulse area except for a fixed insertion loss. The threshold level for the transmitted pulse is fixed and can only be changed via ground command. The detailed calibration between the energy counter output and the actual transmitted laser pulse energy can is given in [10], which also includes the environmental effects such as the temperatures of the laser, the photodiode, and the related electronics.

The laser pulse energy is a function of laser temperature and is also expected to slowly decrease over the laser lifetime [9]. Since the laser pulse energy and width are correlated, the transmitted laser pulsewidth can be inferred from the pulse energy by the relationship [10]

$$FWHM_l = 326.62 \times (\hat{E}_l)^{-0.95} \tag{6}$$

where $\hat{E}_l$ is the transmitted laser pulse energy in mJ.

3 MOLA Receiver Functional Description

The basic MOLA measurement timing diagram is shown in Figure 3 and a simplified receiver block diagram is shown in Figure 4. Details of the instrument optical system design is given by Ramos-Izquierdo et al [11]. A detailed analysis of the characteristics of the laser altimeter receiver is given by Sun et al [12].

The transmitted laser pulses are detected by the start pulse detector, which consists of a photodiode, a lowpass filter, a threshold crossing detector, and a pulse energy counter. The receiver contains a Si avalanche photodiode (APD), a parallel bank of four electrical filters, and a time interval unit (TIU) [13]. The four receiver lowpass filters are 5 pole Bessel design and their 3dB bandwidths and impulse response width (full width at half maximum-FWHM) are all listed in Table 4. The filter impulse response pulse shape is closely approximated by a Gaussian function. When the received signal pulse shape is also Gaussian, the receiver channel with the closest impulse response will have the highest signal to noise ratio (SNR) at the filter output.

The receiver threshold levels are automatically adjusted by an algorithm in the flight computer to maintain a false alarm rate of approximately 1% per channel within the range gate [14]. The threshold settings determines the minimum SNR required for each receiver channel to be triggered. More than one channel may be triggered by the same echo pulse. However, only the channel with the shortest propagation delay stops the TIU and has its measurement results reported to ground. As shown in Figure 3, the round trip time-of-flight of the laser pulses measured at the start and echo pulse centroid points can be calculated from

$$T_{opt} = \frac{N}{f} + \Delta t_0 - \Delta t_1 - \tau_{le}(0) + \tau_f(0) + \tau_{le}(i) - \tau_f(i) - \tau_d(i). \qquad (7)$$

In Figure 3 and Eq. (7),

N:	TIU counter output (counts),
f:	TIU clock frequency (Hz),
Δt_0:	start interpolator reading (seconds),
Δt_1:	stop interpolator reading (seconds),
i:	$i = 0$ represents the start pulse channel,
	$i = 1, 2, 3, 4$ represent the receiver channel which triggered,
$\tau_{le}(i)$:	time from leading edge threshold crossing,

to the pulse centroid,

$\tau_f(i)$: lowpass filter signal propagation delay,

$\tau_d(i)$: receiver circuitry and cable delay,

$A_y(i)$: pulse area between the pulse threshold crossings (volts·sec),

$W_y(i)$: pulsewidth at threshold crossings (sec),

$y(i)$: effective threshold levels (volts).

4 Laser Pulse Time-of-flight Measurement

The time of flight measurements utilize the crystal oscillator, time interval unit, and start and stop interpolators. The timing offset due to leading edge triggering of the time interval unit can be compensated for by using the measured echo pulsewidth, pulse energy, and threshold level.

4.1 The TIU Counter and the Time Base

The time interval unit (TIU) consists of a simple binary counter which registers the total number of clock pulses from the threshold crossings of the transmitted (start) pulse to the echo pulse. The clock consists of an oven-controlled crystal oscillator (OCXO) at 100 MHz which is stable to 10^{-10} over hours [15].

MOLA also time-stamps the laser triggering pulse every 140 laser shots in reference to the spacecraft time base with a resolution of 1/256 second. The laser triggering pulses are generated by dividing the MOLA clock. There is a $189.3 \pm 0.5 \mu s$ delay from laser trigger pulse to the laser pulse emission time. The spacecraft clock is closely monitored on Earth through the spacecraft communication link carrier frequency. The primary purpose of the laser pulse time stamps is to use with the spacecraft orbit and pointing angle data to determine the location of each laser footprint on the Mars surface.

The laser pulse time stamps also serve to monitor the long term aging and drift of the MOLA clock frequency over time during flight. The MOLA clock period can be estimated by averaging the time intervals of adjacent time stamps and dividing by the number of clock cycles elapsed between time stamps. The relative estimation error is bounded by the MOLA time stamp resolution, 1/256 seconds, divided by the total integration time over which the estimation is performed. As an example, it will take one hour and

10 minutes integration time to achieve a relative estimation error less than 2.5ns over a 400km round trip time, or $< 9.7 \times 10^{-7}$. A more accurate clock frequency estimation algorithm has been developed by Neumann [16] in which the quantization errors of the time stamps were assumed as independent and uniformly distributed random variables and a maximum likelihood estimator is used. The resultant clock frequency is given in Table 1.

4.2 Start and Stop Time Interpolators

To improve the range resolution, the MOLA timing unit includes interpolators for the start and the stop pulses. These determine the threshold crossing times to about 1/4 of a clock period. The interpolators return their measurement as a 2 bit index of the start or the stop pulse threshold crossing time relative to the next clock tick. Due to the asymmetries in delays of the timing circuit, the actual interpolator values were not exactly 1/4 clock periods. The actual values were measured in the flight altimeter electronics subsystem tests, and their values are given in Tables 2 and 3.

4.3 Filter Delays

The start detector lowpass filter is a 3 pole Bessel lowpass filter. Its bandwidth and other parameters are listed in Table 4 where it is designated as Channel 0. The receiver lowpass filters are used to maximize the receiver probability of detection given the uncertainties in the echo pulse width and energy. Since all the filters have a 5 pole Bessel lowpass design, the filter propagation delays are given by $\tau_f(i) \approx 1.10\times$ FWHM(i). The receiver lowpass filter characteristics are listed in Table 4 as Channels 1-4.

4.4 Corrections for Leading Edge Timing

The threshold crossing times of the start and stop pulses depends on the threshold level and the shape of the output pulse from the filter. To obtain a unbiased time-of-flight estimate, the optical pulse centroid should be used as the timing point on both pulses. For symmetric pulses, the centroid point is equivalent to the pulse midpoint time. Therefore, the leading edge timing correction is

$$\tau_{le}(i) = \frac{W_y(i)}{2} \tag{8}$$

where $W_y(i)$ is the measured width of the pulse in Channel i.

The timing correction for the start pulse is almost a constant because the laser pulse energy and width change relatively slowly over time and the SNR at the input to the start discriminator is high. Since the start filter impulse response is much wider than the laser pulse, the start pulsewidth is dominated by the filter and is relatively insensitive to the laser pulsewidth variation. For example, if the transmitted laser pulsewidth varies from 8 to 11 ns, the start pulsewidth output from the filter changes from 31.0 to 32.0 ns and the correction for the leading edge timing is < 0.5 ns.

4.5 Instrument Time Bias

In addition to the filter propagation delays, there is also an instrument time bias due to electronics circuitry and cables. The instrument time bias was determined from a series of near range measurements during ground tests and it was taken to be the extrapolated time offset at zero range. The resultant range biases of the entire instrument with leading edge timing and the measured pulsewidth at threshold crossings are given in Table 5. The unknown receiver delays can be solved using Eq.(7) with $T_{opt} = 0$ and $\tau_{le} = W_y(i)/2$. Since the width of the laser pulses was nearly constant in those tests, the start pulsewidth can be approximated by the FWHM pulsewidth which is given by

$$W_y(0) \approx FWHM(0) = \sqrt{FWHM_f(0)^2 + FWHM_l^2} \tag{9}$$

where $FWHM(0)$ is the filter impulse response pulsewidth given in Table 4 and the transmitted laser pulsewidth is $FWHM_l \approx 8.0ns$. The resultant receiver delay times are given in Table 5. The estimated echo pulsewidth, $W_y(i)$, i=1,...,4, are also listed. For convenience, the delays of the start pulse detection circuit was set to zero and its effect was accounted for in the receiver channels.

5 Pulsewidth, Area, and Energy Measurement

In addition to time of flight measurement, MOLA receiver also measures the pulsewidth and area of the filtered echo pulse at the threshold crossings. The measured width and area depend on the threshold level, the filter impulse response, and the echo pulse shape. The actual echo pulsewidth and energy can be calculated given these measurement and system parameters.

5.1 Pulsewidth at the Threshold Crossings

The pulsewidth measured by MOLA is accurately approximated by

$$W_y(i) = a_W(i)[l(i) - b_W(i)], \quad i = 1, 2, 3, 4. \tag{10}$$

Here $l(i)$ is the pulsewidth count reported by MOLA for Channel i, and $b_W(i)$ and $a_W(i)$ are the count offset and the conversion factor for Channel i, respectively. These constants are given in Table 6 and they were determined during the altimeter electronics tests when an electrical pulse was used as the input signal.

Due to the speed limitations of the electronics, the measured pulsewidth from Channel 1 output deviates from a linear relationship for short widths. The flight MOLA receiver characterization showed that the Channel 1 pulsewidth measurement can be approximated by two linear equations which intersect at 17 ns (12 counts) pulsewidth, as listed in Table 6. The maximum number of 63 counts in the pulsewidth counters set the upper limits of the linear dynamic range to about 200, 450, 750, and 1600 ns for Channels 1-4 respectively.

5.2 Pulse Area between Threshold Crossings

The pulse area measured by MOLA is accurately approximated by

$$A_y(i) = a_A(i)[\, m(i) - b_A(i)] \tag{11}$$

where $A_y(i)$ is the pulse area in volts·ns, $m(i)$ is the MOLA reported pulse area count, and $a_A(i)$ and $b_A(i)$ are the conversion factors and the count offsets for Channel i. The conversion factors and the count offsets were determined during the altimeter electronics subsystem tests when a pulse generator was used as the input signal source. The results are listed in Table 6.

5.3 Channel Gain and Threshold Scaling Factors

The threshold level settings reported by MOLA are those directly applied to the discriminators. In the actual instrument, each channel has a different gain in order to optimize the receiver dynamic range. To relate the reported thresholds to the effective thresholds shown in Figures 2 and 3, their values have to be scaled by the power splitter loss and the voltage gain factors for each channel. The resulting effective threshold levels are given by

$$y(i) = a_{thre}(i)th_i \tag{12}$$

where th_i is the actual MOLA threshold voltage levels at Channel i as reported in the data packet and $a_{thre}(i)$ is the scaling factor listed in Table 6.

5.4 Solving for the Pulse Parameters

The laser echo pulsewidth and energy before the threshold crossing circuits may be determined if the pulse shape is known. For a Gaussian input and Gaussian filter impulse response, the output pulse shapes from the lowpass filter are also Gaussian and can be written as

$$f_i(t) = \frac{A}{\sqrt{2\pi}\sigma_r(i)} e^{-\frac{t^2}{2\sigma_r(i)^2}}. \tag{13}$$

Here A is the pulse area and $\sigma_r(i)$ is the rms width, and we have set the time origin to zero for convenience. The MOLA measured pulsewidth, $W_y(i)$, and the threshold level, $y(i)$, are related by

$$y(i) = f_i(W_y(i)/2) = \frac{A}{\sqrt{2\pi}\sigma_r(i)} e^{-\frac{(W_y(i)/2)^2}{2\sigma_r(i)^2}}. \tag{14}$$

The MOLA measured pulse area between the threshold crossings is given by

$$\begin{aligned} A_y(i) &= \int_{-W_y(i)/2}^{W_y(i)/2} \frac{A}{\sqrt{2\pi}\sigma_r(i)} e^{-\frac{t^2}{2\sigma_r(i)^2}} dt \\ &= A\, erf(\frac{W_y(i)/2}{\sqrt{2}\sigma_r(i)}) \end{aligned} \tag{15}$$

where $erf(x)$ is the standard error function.

We can solve for the rms pulsewidth by taking the ratio of Eqs. (15) and (14) and eliminating A, yielding

$$\frac{A_y(i)}{y(i)} = \frac{W_y(i)\sqrt{\pi}}{2} \frac{erf(\frac{W_y(i)/2}{\sqrt{2}\sigma_r(i)})}{\frac{W_y(i)/2}{\sqrt{2}\sigma_r(i)}\ e^{-(\frac{W_y(i)/2}{\sqrt{2}\sigma_r(i)})^2}}. \tag{16}$$

This can be simplified to

$$\frac{A_y(i)}{y(i)W_y(i)} = z(\frac{W_y(i)/2}{\sqrt{2}\sigma_r(i)}). \tag{17}$$

Here the function

$$z(x) \equiv \frac{\sqrt{\pi}}{2}\frac{erf(x)}{xe^{-x^2}} \tag{18}$$

is a monotonically increasing function with a unique inverse function.

The rms pulsewidth $\sigma_r(i)$ can be solved for as

$$\sigma_r(i) = \frac{W_y(i)/2}{\sqrt{2}}[z^{-1}(\frac{A_y(i)}{y(i)W_y(i)})]^{-1}. \tag{19}$$

Similarly, the full pulse energy can be solved as

$$A = A_y(i)\frac{1}{erf(z^{-1}(\frac{A_y(i)}{y(i)W_y(i)}))} \tag{20}$$

The inverse function $z^{-1}(x)$ can be obtained by standard numerical techniques or curve fitting. One example of a polynomial fit over $0.2 < x < 2.2$ is given by

$$\begin{aligned} z_c^{-1}(x) &= 0.1716 + 4.9319[log(x)] - 11.693[log(x)]^2 + 18.886[log(x)]^3 \\ &\quad -16.696[log(x)]^4 + 7.4269[log(x)]^5 - 1.2997[log(x)]^6 \end{aligned} \tag{21}$$

Figure 5 shows a plot of the original $z^{-1}(x)$ and the curve fit given above.

5.5 Echo Laser pulsewidth and Energy

The results above can be used to solve for the energy and width of the MOLA echo pulses. Since the receiver electrical bandwidth is primarily limited by the lowpass filter in each channel, the detected pulse shape can be assumed unchanged up to the filters. The lowpass filters cause pulse spreading but preserve the pulse energy. For Gaussian received pulse shape and Gaussian filter impulse responses, the lowpass filter output is also Gaussian. Therefore, and the rms pulsewidth of the echo laser pulse can be solved for as

$$\sigma_{opt} = \sqrt{\sigma_r(i)^2 - [\sigma_f(i)]^2} \tag{22}$$

Here the rms pulsewidths of the filters are related to the FWHM impulse response pulsewidth by

$$\sigma_f(i) = \frac{FWHM(i)}{2\sqrt{2ln(2)}} \tag{23}$$

and their values are given in Table 4.

Finally, the optical energy of the echo pulse can be calculated by dividing the pulse area by the detector assembly responsivity, yielding,

$$E_r = \frac{A}{R_{det}} \tag{24}$$

The detector responsivity was measured to be $R_{det} = 1.26 \times 10^8$ V/watt at room temperature in preflight testing. The received pulse energy in detected photons (i.e. photoelectrons) can be calculated as

$$N_{pe} = \frac{\eta E_r}{hc/\lambda} \tag{25}$$

where η is the photodetector quantum efficiency, h is Planck's constant, and λ is the laser wavelength.

6 Measurement Error Analysis

The error in the MOLA measurements depend on the signal energy, pulse width, background level, and detector noise, and they are summarized in this section.

6.1 Variance of Time-of-Flight Measurement

The time-of-flight measured by MOLA can be expressed as

$$T_{opt} = t_r + W_y/2 + T_o + \epsilon_Q \tag{26}$$

where t_r is the receiver threshold crossing times, W_y is the measured pulse width at the threshold crossings, T_o is a constant timing offset which accounts for all the filter and electronics delays and the start pulse centroid corrections, and ϵ_Q is the quantization error due to the limited TIU and pulse width counter step resolution. The pulse width is given by $W_y = t_f - t_r$ with t_f the threshold crossing time at the pulse trailing edge. To simplify the notation, the index of triggering channel number is omitted in this section.

The variance of the measured laser pulse time-of-flight can be written as

$$\begin{aligned} Var(T_{opt}) &= Var(\frac{t_r + t_f}{2}) + Var(\epsilon_Q) \\ &= \frac{1}{4}\left[Var(t_r) + Var(t_f)\right] + \frac{\Delta t_0^2}{12} + + \frac{\Delta t_1^2}{12} + \frac{\Delta W^2}{12} \end{aligned} \quad (27)$$

where Δt_0, Δt_1, and ΔW are the step sizes of the start and stop interpolators and the pulse width counters. For MOLA, $\Delta t_0 \approx \Delta t_1 \approx 2.5ns$, and ΔW is same as $a_W(i)$ in Table 6.

The variances of the threshold crossing times, $Var(t_r)$ and $Var(t_f)$, can be determined by adapting the derivation by Davidson and Sun [17]. The signal output from the receiver lowpass filter can be written as

$$x(t) = s(t) + n(t) \quad (28)$$

with $s(t) = < x(t) >$ and $n(t) = x(t) - < x(t) >$. The average signal can be approximated by the first two terms of its Taylor expansion about the average leading edge threshold crossing time, as

$$s(t) \approx s(T_r) + s'(T_r)(t - T_r) \quad (29)$$

with $T_r = < t_r >$ the average threshold crossing time. Substituting Eq. (29) into (28) and let $t = t_r$,

$$x(t_r) \approx s(T_r) + s'(T_r)(t_r - T_r) + n(t_r). \quad (30)$$

By definition, $x(t_r) \equiv s_{th}$ with s_{th} the threshold level. Averaging both sides of (30) yields $s(T_r) \equiv s_{th}$ and,

$$s'(T_r)(t_r - T_r) + n(t_r) \approx 0 \quad (31)$$

The variance of threshold crossing time can now be written as,

$$Var(t_r) = < (t_r - T_r)^2 > \approx \frac{Var[n(T_r)]}{[s'(T_r)]^2}. \quad (32)$$

Note $Var(t_r)$ is a function of the average threshold crossing time which is a function of the threshold level and the signal pulse amplitude and shape.

To obtain the derivatives of the signal, the output from the receiver lowpass filter can be modeled as a filtered Poisson point process [18]. The average signal in terms of the detected photons/sec can be written as

$$s(t) = N_{pe} \int_{-\infty}^{\infty} h_f(\tau) p(t - \tau) d\tau \tag{33}$$

where $h_f(t)$ is the receiver lowpass filter impulse response and $p(t)$ is the normalized received optical pulse shape, which satisfy $\int_{-\infty}^{\infty} h_f(t)dt = 1$ and $\int_{-\infty}^{\infty} p(t)dt = 1$. The derivative of the signal from the lowpass filter can be written as

$$s'(t) = N_{pe} \int_{-\infty}^{\infty} h_f(\tau) p'(t - \tau) d\tau \tag{34}$$

Eq. (34) can be evaluated by assuming the received optical signal pulse shape and the lowpass filter impulse response are both Gaussian with zero mean and rms pulse width of σ_{opt} and σ_f, respectively. The output pulse shape under this assumption is also Gaussian with rms pulse width $\sigma_r = \sqrt{\sigma_{opt}^2 + \sigma_f^2}$.

For Gaussian pulses, the average threshold crossing time is given by

$$T_r = -2\sigma_r \; ln \left(\frac{s_{th}}{N_{pe}/\sqrt{2\pi}\sigma_r} \right) \tag{35}$$

where s_{th} is given in terms of detected photons per second. Note the ratio of the threshold crossing time to the rms pulse width depends only on the ratio of the threshold level to the pulse amplitude. The threshold level s_{th} is related to the effective voltage threshold level of Figure 3 by

$$s_{th} = \frac{y}{R_{det} \frac{hc/\lambda}{\eta_d}} \tag{36}$$

The total noise variance in Eq. (32) can be written as the sum of the variances of shot noise due to the detected signal, the background radiation, and the detector dark count, and the preamplifier noise, i.e.,

$$Var[n(t)] = Var[n_{sig}(t)] + Var(n_{bg}) + Var(n_{dk}) + Var(n_{amp}). \tag{37}$$

The variance of the shot noise due to the signal is given by [18]

$$Var[n_{sig}(t)] = F N_{pe} \int_{-\infty}^{\infty} h_f^2(\tau) p(t - \tau) d\tau \tag{38}$$

where F is the detector excess noise factor defined as $F = <g_d>^2 \; / <g_d^2>$ with g_d the photodetector multiplication gain. MOLA used a Si APD as the photodetector, the excess noise factor is given by [19],

$$F = k_{eff}G + (2 - \frac{1}{G})(1 - k_{eff}) \tag{39}$$

where k_{eff} is the ionization coefficient ratio and $G = <g_d>$ is the average APD gain. For the detector used in MOLA, $k_{eff} \approx 0.008$ and $G \approx 120$.

The variance of shot noise due to the background light can be written as [20] [21]

$$Var(n_{bg}) = 2F\frac{\eta_d}{hc/\lambda}P_{bg}B_n \tag{40}$$

where P_{bg} is the background light power onto the detector and B_n is the one sided filter noise bandwidth given by

$$B_n = \frac{1}{2}\int_{-\infty}^{\infty} h_f^2(\tau)d\tau. \tag{41}$$

The background light power on the detector can be calculated as

$$P_{bg} = I_s \cdot \Delta\lambda\frac{\theta_{FOV}^2}{4}r_sA_r\tau_r \tag{42}$$

where $I_s = 0.311 W/m^2nm$ is the solar irradiance at Mars, $\Delta\lambda$ is the receiver optical bandwidth, θ_{FOV} is the receiver FHWM field of view, and τ_r, r_s, and A_r are the same as in (4).

Similar to the signal shot noise, the detector dark current shot noise can be written as

$$Var(n_{dk}) = 2F\frac{I_{dk}}{q}B_n \tag{43}$$

where I_{dk} is the detector bulk dark current and q is the electron charge.

The variance of the detector preamplifier noise can be written as

$$Var(n_{amp}) = \frac{I_{amp}^2B_n}{q^2G^2} \tag{44}$$

with I_{amp}^2 the equivalent input noise current density of the preamplifier in A^2/Hz.

Substitute Eqs. (38) through (44) into (37) and then Eqs. (34) and (37) into (32), the variance of the leading edge threshold crossing time can be written as

$$Var(t_r) = \frac{FN_{pe}\int_{-\infty}^{\infty} h_f^2(\tau)p(T_r-\tau)d\tau + 2F[\frac{\eta_d}{hc/\lambda}P_{bg} + \frac{I_{dk}}{q}]B_n + \frac{I_{amp}^2}{q^2G^2}B_n}{\left[N_{pe}\int_{-\infty}^{\infty} h_f(\tau)p'(T_r-\tau)d\tau\right]^2} \tag{45}$$

Under the assumption of Gaussian pulse shapes, the average threshold crossing time at the rising and trailing edge are symmetric, $T_f = -T_r$, $N_{sig}(T_r) = N_{sig}(T_f)$, $s'(T_f) = -s'(T_r)$, and

$$Var(t_r) = Var(t_f). \tag{46}$$

The variance of the time-of-flight measurement can be obtained by substituting (45) and (46) into (27).

A lower bound on variance of the time-of-flight measurement has been derived by Gardner [6]

$$Var(T_{opt}) \geq \frac{\sigma_{opt}}{\sqrt{N_{pe}/F}} \tag{47}$$

This lower bound can be achieved by recording the entire waveform and calculate the centroid under no background illumination and amplifier noise.

6.2 Pulse Width Measurement Error

The Variance of the pulse width measurement at the threshold crossings is given by

$$Var(W_y) = Var(t_r - t_f) + \frac{\Delta W^2}{12} = Var(t_r) + Var(t_f) + \frac{\Delta W^2}{12}. \tag{48}$$

Note the variance given above is for the pulse width directly measured at the threshold crossings. The error in the calculated rms pulse width given in (19) is in general larger.

6.3 Variance of the Pulse Area Measurement Error

The integration process for the pulse area measurement can be treated as the sampled output of a box-car integrator lowpass filter. The impulse response of the box-car integrator can be written as

$$h_A(t) = \begin{cases} 1, & T_r \leq t \leq T_f \\ 0, & otherwise \end{cases} \tag{49}$$

The output of the integrator can be modeled as a filtered Poisson random point process [18] with the filter being the cascade of the box-car integrator and the lowpass filter of channel under consideration, i.e.,

$$h_{fA}(t) = \int_{-\infty}^{\infty} h_f(t-u)h_A(u)du = \int_{T_r}^{T_f} h_f(t-u)du \tag{50}$$

The measured pulse area in number of photoelectrons is equal to $N_{pe}h_{fA}(0)$. The pulse area in volt·seconds is given by

$$A_y = \left[R_{det}\frac{hc/\lambda}{\eta_d}\right] N_{pe} \cdot h_{fA}(0) \tag{51}$$

where the term in brackets represent a conversion factor from the detected photons/sec to volts at the output of the detector assembly.

The variance of pulse area measurement at the threshold crossings can be written, similarly to (38), as

$$Var(A_y) = \left[R_{det}\frac{hc/\lambda}{\eta_d}\right]^2 FN_{pe}\int_{-\infty}^{\infty}[\int_{T_r}^{T_f} h_f(\tau-u)du]^2 p(-\tau)d\tau + \frac{\Delta A^2}{12} \tag{52}$$

where ΔA is the pulse area counter resolution which is the same as $a_A(i)$ in Table 6.

If the received optical signal pulse width is much wider than the filter impulse response, $\int_{T_r}^{T_f} h_f(\tau-u)du \approx 1$ for $T_r < \tau < T_f$, and

$$\begin{aligned} Var(A_y) &\approx \left[R_{det}\frac{hc/\lambda}{\eta_d}\right]^2 FN_{pe}\int_{T_r}^{T_f} p(-\tau)d\tau + \frac{\Delta A}{12} \\ &\approx \left[R_{det}\frac{hc/\lambda}{\eta_d}\right]^2 FN'_{pe} + \frac{\Delta A}{12} \end{aligned} \tag{53}$$

with N'_{pe} the number of photoelectrons integrated between the threshold crossings.

6.4 Numerical Examples

Table 1 lists all the MOLA system parameter values needed to calculate the measurement errors. Figures 6 through 8 show plots of the instrument performance, all for Channel 1, based on the analysis given in this paper.

Figure 6 shows the rms ranging error vs. received signal level for various ground target slopes and at a nominal spacecraft altitude of 400km. The ranging error increases rapidly at low signal level because the threshold level is near the peak of the pulse and the receiver channel is operating with a shallow slope and a poor SNR. The daytime ranging error is better than nighttime at high signal level because the threshold level for daytime is roughly twice as high and closer to the optimal level. Note the threshold level is set to maximize the detection probability rather than to minimize the ranging error. The receiver sensitivity and performance of Channels 2-4 are in general better than Channel 1 for higher slope surface. Figure 6 also shows the receiver quantization error is the dominating factor for targets with small slopes. More detailed analysis also shows the error floor due to the receiver dark noise alone is about 30% the quantization error. The lower bound given by Eq. (47) is roughly one third that of leading edge threshold crossing detection error given by (45).

Figure 7 shows a plot of rms ranging error vs. normalized threshold level for 1, 3, 10, and 30 degree slopes at a spacecraft altitude of 400km. It shows the range error is relatively insensitive to the threshold level as long as the it is between 20 to 80% of the peak pulse amplitude but increase rapidly as the threshold is near the top or the bottom of the pulse waveform due to the low slopes of the signal waveform at the threshold crossing.

Figure 8 shows the ranging error vs. spacecraft altitude or the ranging distance. It is particularly useful for estimating MOLA receiver performance during operation before the MGS spacecraft reached its final circular orbit around mars. Note both the signal level and the received pulsewidth changes with the spacecraft altitude. It again shows the ranging error increases very rapidly as the received signal level is near the detecting limit and the threshold level is approaching the peak of the pulse waveform. The echoes from surface with larger slopes may still be detected by Channels 2-4.

7 Acknowledgments

We would like to thank John F. Cavanaugh, Richard B. Katz, James R. Baker, Catherine A. Long, James N. Caldwell, Alan T. Lukemire, J. Bryan Blair, Jan F. McGarry, and Gregory C. Elman for their many expert contributions in the Mola electronics system and Ronald B. Follas and James C. Smith for their guidance of the MOLA instrument development.

References

[1] M. T. Zuber, D. E. Smith, S. C. Soloman, D. O. Muhleman, J. W. Head, J. B. Garvin, J. B. Abshire, and J. L. Bufton, 'Mars Observer Laser Altimeter investigation,' *J. Geophys. Res*, Vol. 97, pp. 7781-7797, May 1992.

[2] D. E. Smith, M. T. Zuber, H. V. Frey, J. B. Garvin, J. W. Head, D. O. Muhleman, G. H. Pettengill, R. J. Phillips, S. C. Solomon, H. J. Zwally, W. B. Banerdt, and T. C. Duxbury, 'Topography of the northern hemisphere of Mars from the Mars Orbiter Laser Altimeter,' *Science*, Vol. 279, pp. 1686-1692, Mar. 1998.

[3] M. T. Zuber, D. E. Smith, S. C. Solomon, J. B. Abshire, R. S. Afzal, O. Aharonson, K. Fishbaugh, P. G. Ford, H. V. Frey, J. B. Garvin, J. W. Head, A. B. Ivanov, C. L. Johnson, D. O. Muhleman, G. A. Neumann, G. H. Pettengill, R. J. Phillips, X. Sun, H. J. Zwally, W. B. Banerdt, and T. C. Duxbury, 'Observation of the north polar region of Mars from the Mars Orbiter Laser Altimeter,' *Science*, Vol. 282, pp. 2053-2060, Dec. 1998.

[4] A. A. Albee, F. D. Palluconi, and R. E. Arvidson, 'Mars Global Surveyor Mission: overview and status,' *Science*, Vol. 279, pp. 1671-1672, Mar. 1998.

[5] E. R. Cohen and B. N. Taylor, 'The fundamental physical constants,' *Physics Today*, pp. BG7-BG14, Aug. 1997.

[6] C. S. Gardner, 'Target signatures for laser altimeters: an analysis,' *Applied Optics*, Vol. 21, pp. 448-453, Feb. 1982.

[7] C. S. Gardner, 'Ranging performance of satellite laser altimeters, *IEEE Trans Geoscience and Remote Sensing*, Vol. 30, No. 5, pp. 1061-1072, Sept. 1992.

[8] D. D. Rowlands, D.E. Pavlis, F.G. Lemoine, G.A. Neumann, and S.B. Luthcke, 'The use of laser altimetry in the orbit and attitude determination of Mars Global Surveyor,' *Geophysical Research Letters*, Vol. 26, No. 9, pp. 1191-1194, May 1, 1999.

[9] R. S. Afzal, 'The Mars Observer Laser Altimeter: Laser Transmitter,' *Applied Optics*, Vol. 33, No. 15, pp. 3184-3188, 20 May 1994.

[10] R. S. Afzal, 'MOLA-2 laser transmitter calibration and performance,' Technical report, NASA GSFC, Laser Remote Sensing Branch, Greenbelt, MD 20771, Sept. 1997.

[11] L. Ramos-Izquierdo, J. L. Bufton, and P. Hayes, 'Optical system design and integration of the Mars Observer Laser Altimeter,' *Applied Optics*, Vol. 33, No. 3, pp. 307-322, Jan. 1994.

[12] X. Sun, F. M. Davidson, L. Boutsikais, and J. B. Abshire, 'Receiver characteristics of laser altimeters with avalanche photodiode' *IEEE Trans. Aerospace and Electronics Systems*, Vol. 28, No. 1, pp. 268-274, Jan. 1992.

[13] J. B. Abshire, S. S. Manizade, W. H. Schaefer, R. K. Zimmerman, J. S. Citwood, and J. C. Caldwell, 'Design and performance of the receiver for the Mars Observer Laser Altimeter,' *Conference on Laser and Electro-Optics (CLEO'91)*, Paper CFI4, Technical Digest, Optical Society of America, Washington DC, May 1991.

[14] J. F. McGarry, L. K. Pacini, J. B. Abshire, and J. B. Blair, 'Design and performance of an autonomous tracking system for the Mars Observer Laser Altimeter receiver,' *Conference on Laser and Electro-Optics (CLEO'91)*, Paper CThR27, Technical Digest, Optical Society of America, Washington DC, May 1991.

[15] J. R. Vig, *Introduction of Quartz Frequency Standards*, Research and Development Technical Report, CECOM-TR-97-3, June 1997, U.S. Army Communications-Electronics Command (CECOM), Research, Development and Engineering Center, Fort Monmouth, New Jersey 07703-5000.

[16] G. A. Neumann, 'Time on Mars,' internal technical report, NASA GSFC, Code 920, Feb. 1998.

[17] F. M. Davidson and X. Sun, 'Slot clock recovery in optical PPM communication systems with avalanche photodiode photodetectors,' *IEEE Trans. Commun.*, Vol. 37, No. 11, pp. 1164-1172, Nov. 1989.

[18] D. L. Snyder, *Random Point Process*, John Viley & Sons, New York, 1975.

[19] P. P. Webb, R. J. McIntyre, and J. Conradi, 'Properties of avalanche photodiodes' *RCA Review*, Vol. 35, pp. 234-278, June 1974.

[20] R. Gagliardi and S. Karp, *Optical Communication*, Wiley, New York, 1976.

[21] R. G. Smith and S. D. Personick, 'Receiver design for optical fiber communication s,' in *Semiconductor Devices for Optical Communication*, H. Kressel, Ed., Springer-Verlag, New York, 1980, ch. 4.

Table 1: MOLA System Parameters

Symbol	Value	Description
E_{tr}	42 mJ	nominal transmitted laser pulse energy
λ	1064 nm	laser wavelength
$FWHM_l$	8 ns	nominal transmitted laser pulse FWHM
θ_x	0.37 mrad	laser beam full divergence angle at $1/e^2$ point (rms angle $\gamma = \theta_x/4$)
A_r	0.170 m^2	receiver telescope entrance aperture area
θ_{FOV}	0.850 mrad	receiver field of view
τ_r	0.565	receiver optics transmission
$\Delta\lambda$	2.0 nm	receiver optical bandwidth
η_d	0.35	APD quantum efficiency at 1064 nm
G	120	average APD gain
k_{eff}	0.008	APD ionization coefficient ratio
I_{dk}	50 pA	APD bulk leakage current
N_{amp}	$(2.0pA/Hz^{1/2})^2$	preamplifier input noise density
R_{det}	1.26×10^8 V/W	detector assembly responsivity
$\Delta t_0, \Delta t_1$	2.5 ns	TIU timing resolution
f	99.996311MHz $\pm$1.3Hz	master clock frequency before launch
	99.996232MHz $\pm$2.0Hz	master clock frequency as of March 1999

Table 2: Start interpolator bit pattern to time conversion

Start interpolator bit pattern	Median time offset Δt_0 (ns)	Interpolator bin width (ns)
00	1.1	2.2
01	3.6	2.8
10	5.9	1.8
11	8.4	3.2

Table 3: Stop interpolator bit pattern to time conversion

TIU counter reading	Stop interpolator bit pattern	Median time offset Δt_1 (ns)	interpolator bin width (ns)
even	00	0.9	1.8
even	01	3.2	2.8
even	10	5.5	2.6
even	11	8.4	2.6
odd	00	1.4	2.8
odd	01	3.8	2.0
odd	10	6.1	3.0
odd	11	9.2	3.2

Table 4: Lowpass Filter Parameters

Channel i	impulse width $FWHM_f(i)$ (ns)	rms width $\sigma_f(i)$ (ns)	delay $\tau_f(i)$ (ns)	noise BW B_n (MHz)
0	28.3	9.70	23.3	12.6
1	20	8.49	22	16.6
2	60	25.5	66	5.54
3	180	76.4	198	1.85
4	540	229	594	.615

Table 5: Instrument zero range time offsets with leading edge timing at the average day time threshold level and the calculated instrument time bias

Channel i	Leading edge time offset (ns)	Estimated Pulsewidth $W_y(i)$ (ns)	Instrument time bias $\tau_d(i)$ (ns)
1	36.9	40	43
2	54.7	93	43
3	106	230	31
4	343	480	-3

Table 6: Pulse Width and Area Conversion Factors, Count Offsets, and the Threshold Scaling Factors

Channel i	$a_w(i)$ (ns/ct)	$b_w(i)$ (cts)	$a_A(i)$ (Vns/ct)	$b_A(i)$ (cts)	$a_{thre}(i)$
1	3.60 (0.768 if $< 12cts$)	7.4 (-10.5)	0.411	2.3	2.29
2	7.79	5.3	0.434	3.2	1.32
3	13.5	7.1	0.411	6.0	0.763
4	30.6	12.0	0.429	10	0.440

Figure Captions

Figure 1. MGS spacecraft and MOLA.

Figure 2. Illustration of the MOLA measurement geometry. The drawing assumes that the pointing angle, the pointing error, and the surface normal vectors are all in the same plane.

Figure 3. Timing diagram of the MOLA optical and electrical pulses.

Figure 4. Simplified MOLA receiver blok diagram assuming lossless power splitter and filters and unity scaling factor for all receiver channels. The filter characteristics are listed in Table 4.

Figure 5. The original and the polynominal fit of the inverse Z function.

Figure 6. MOLA Channel 1 ranging error vs. received signal level for 1, 3, 10, and 30 degree slope targets.

Figure 7. MOLA Channel 1 Ranging error vs. threshold level normalized with respect to pulse amplitude.

Figure 8. MOLA Channel 1 ranging error as a function of range.